



Hyperbolic Matter

thru

Higgs bundles

elliot kienzle

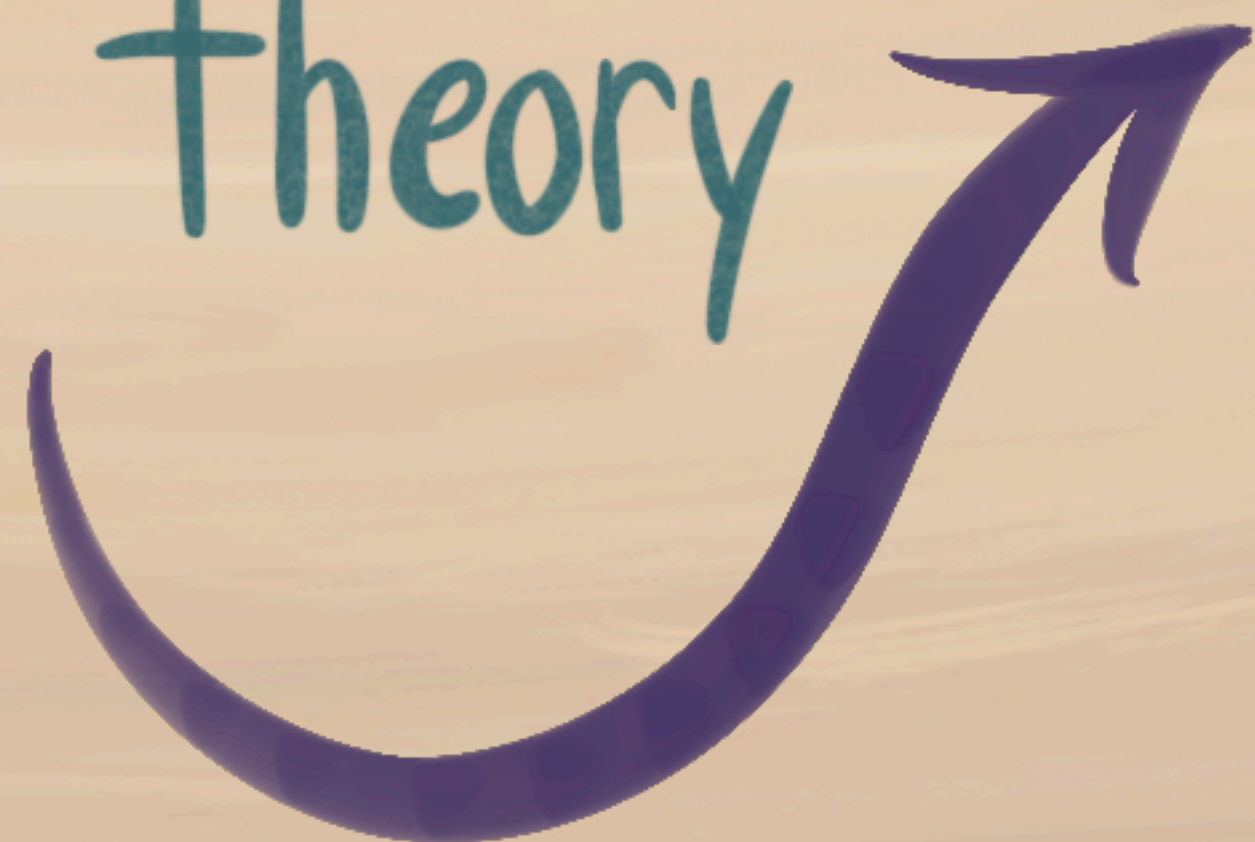
U of Maryland

Outline

Hyperbolic
Band theory

thru

Higgs
Bundles

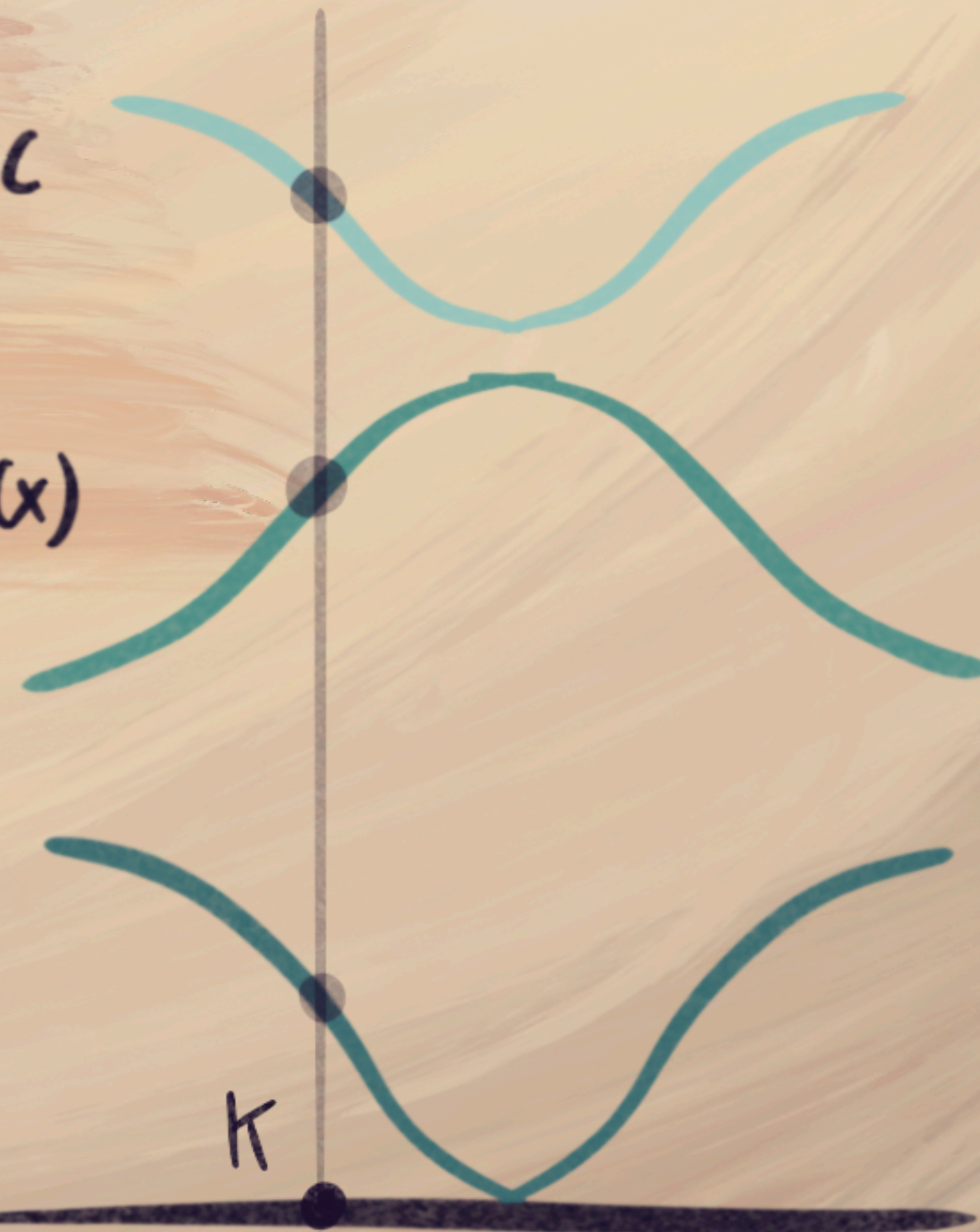


Condensed Matter

Band theory

quasiperiodic states:

$$\Psi(x+T) = e^{i\kappa T} \Psi(x)$$

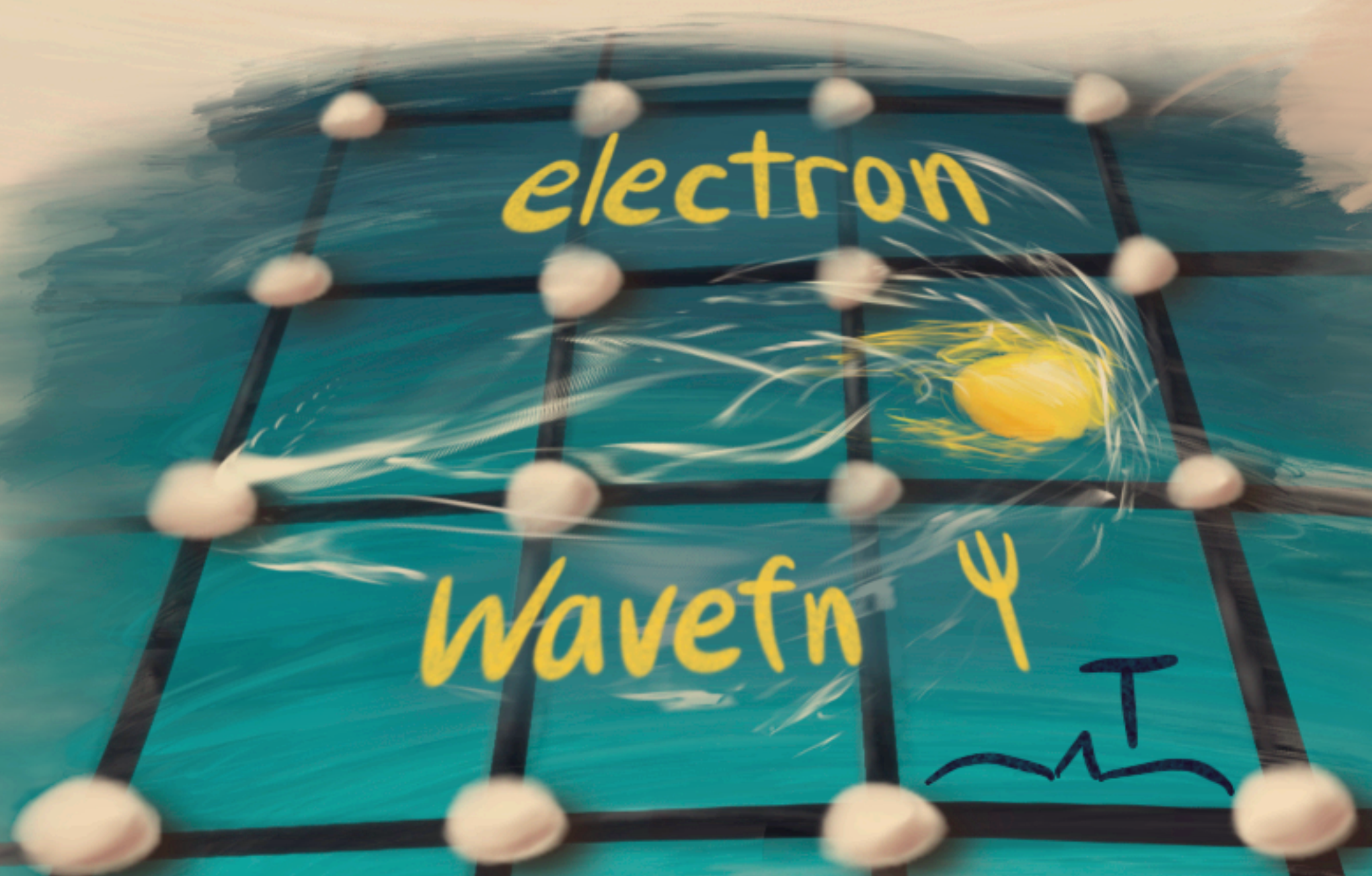


crystal momentum

electron

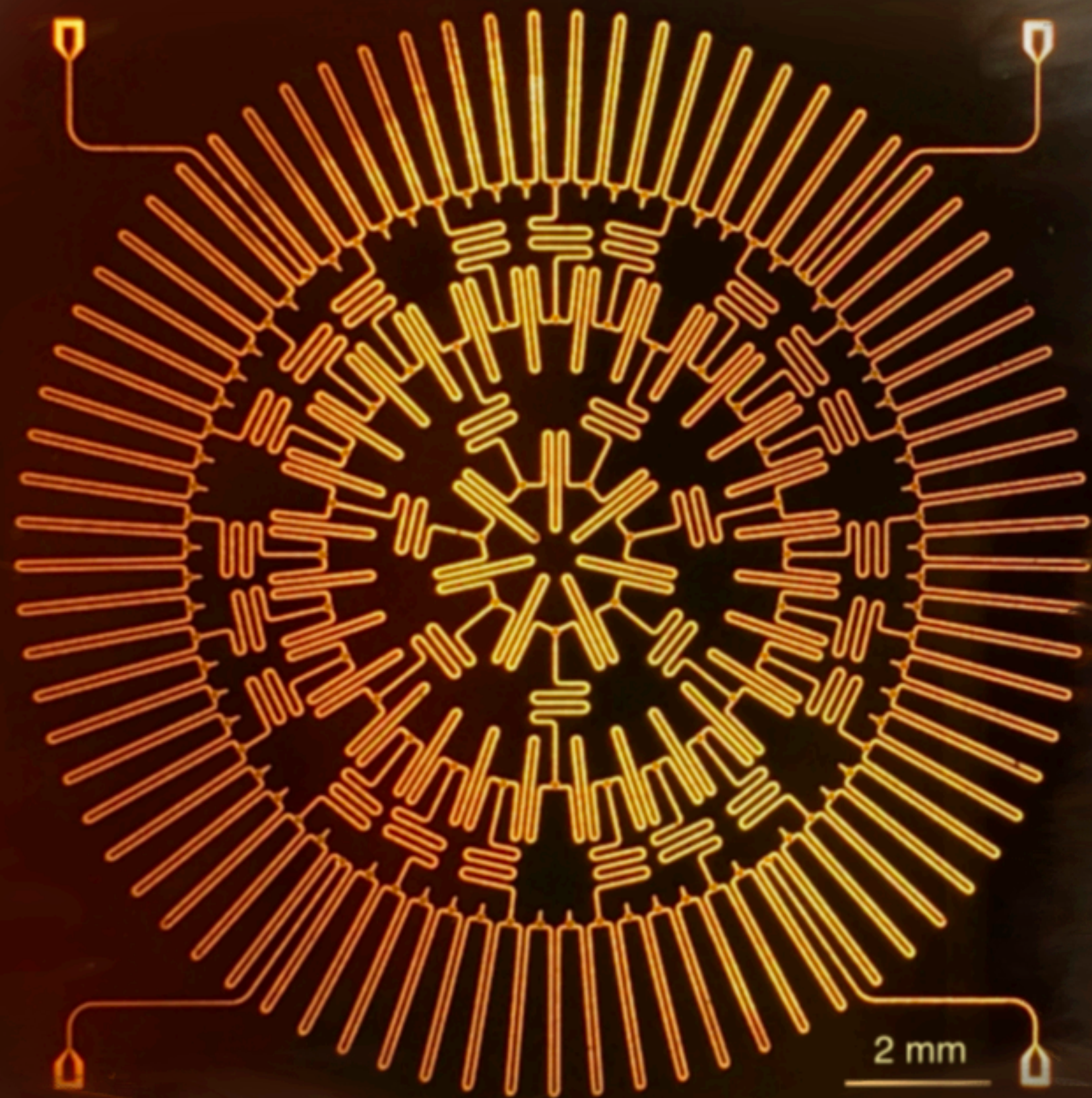
wavefn Ψ

crystal



Hyperbolic Matter

Lattice $\Gamma \subset \text{Isom}(\mathbb{H})$
(constant curvature -1)



kollar et. al. 2018

discrete
model



\Rightarrow Hyperbolic
Band theory!!

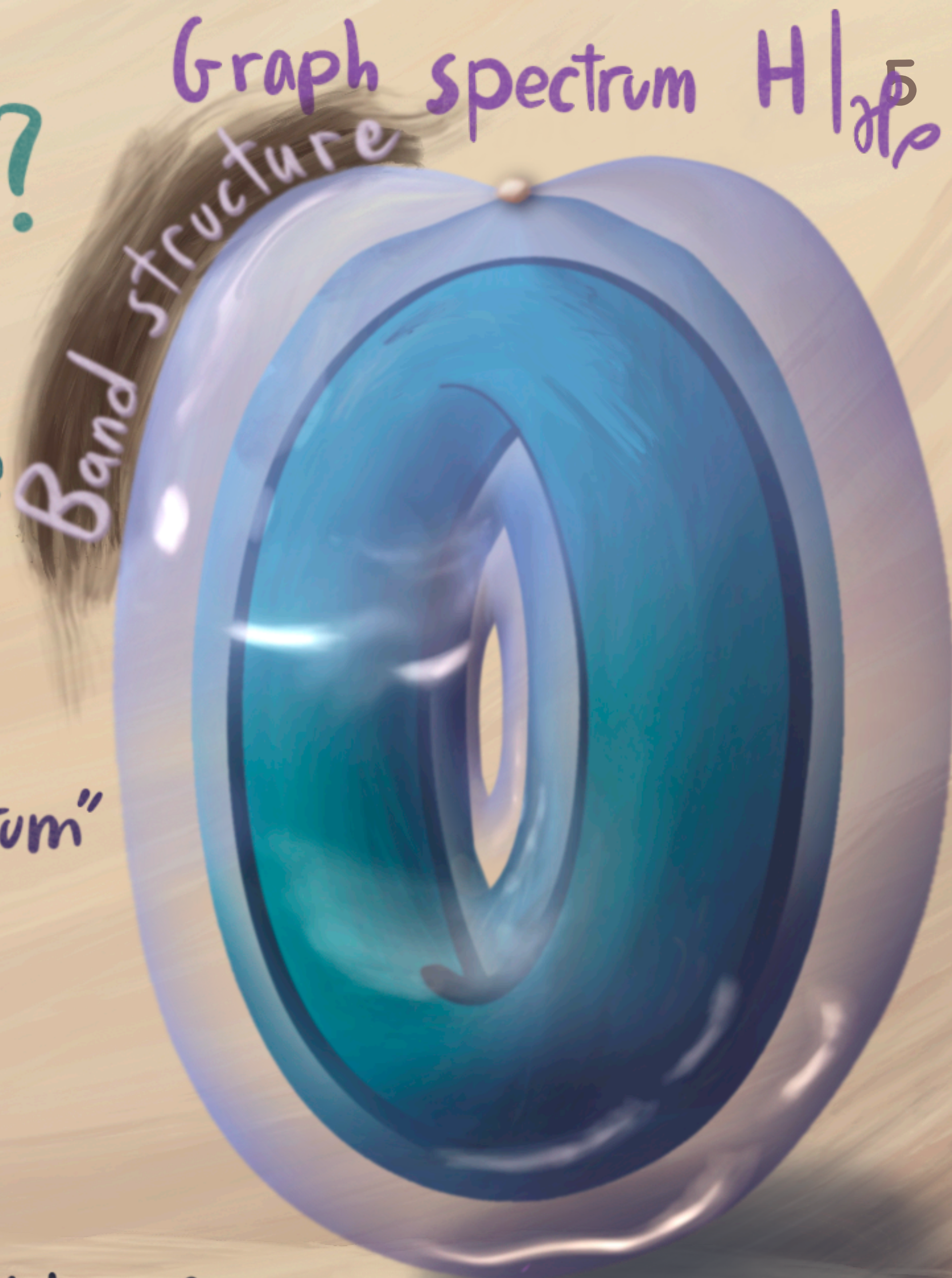
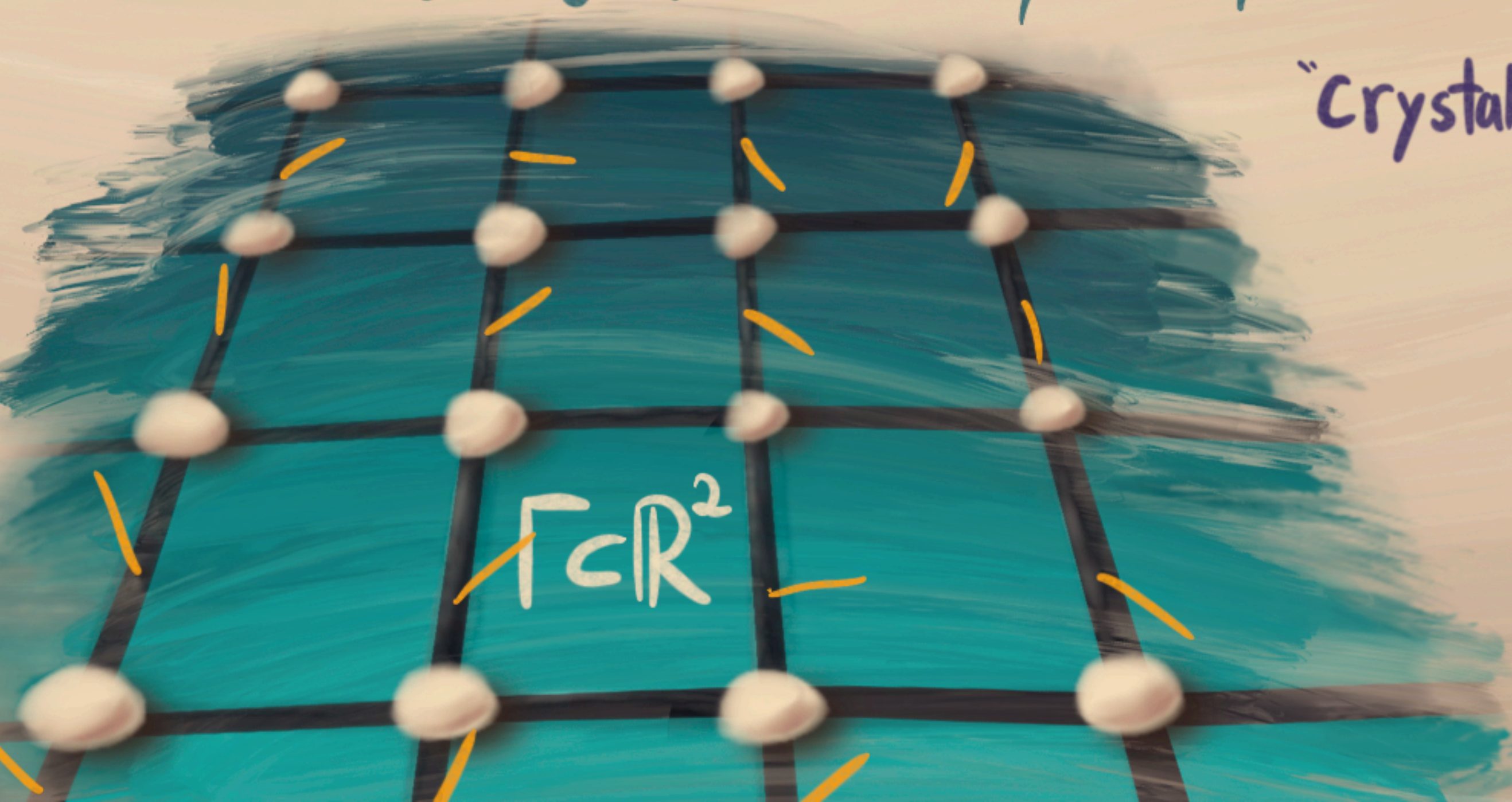
$$H = \Delta + V \stackrel{\text{periodic}}{\curvearrowright} \text{? spectrum?}$$

$$T_{\gamma \in \Gamma} \text{ translation} \quad T_{\gamma_2 \gamma_1} = T_{\gamma_1 + \gamma_2} = T_{\gamma_1} T_{\gamma_2}$$

$$\text{eigenspace: } \mathcal{H}_\rho: \Gamma \rightarrow U(1) = \{ \Psi \mid T_\gamma \Psi = \rho(\gamma) \Psi \}$$

$$HT_\gamma = T_\gamma H \Rightarrow H: \mathcal{H}_\rho \rightarrow \mathcal{H}_\rho$$

"crystal momentum"
 ρ



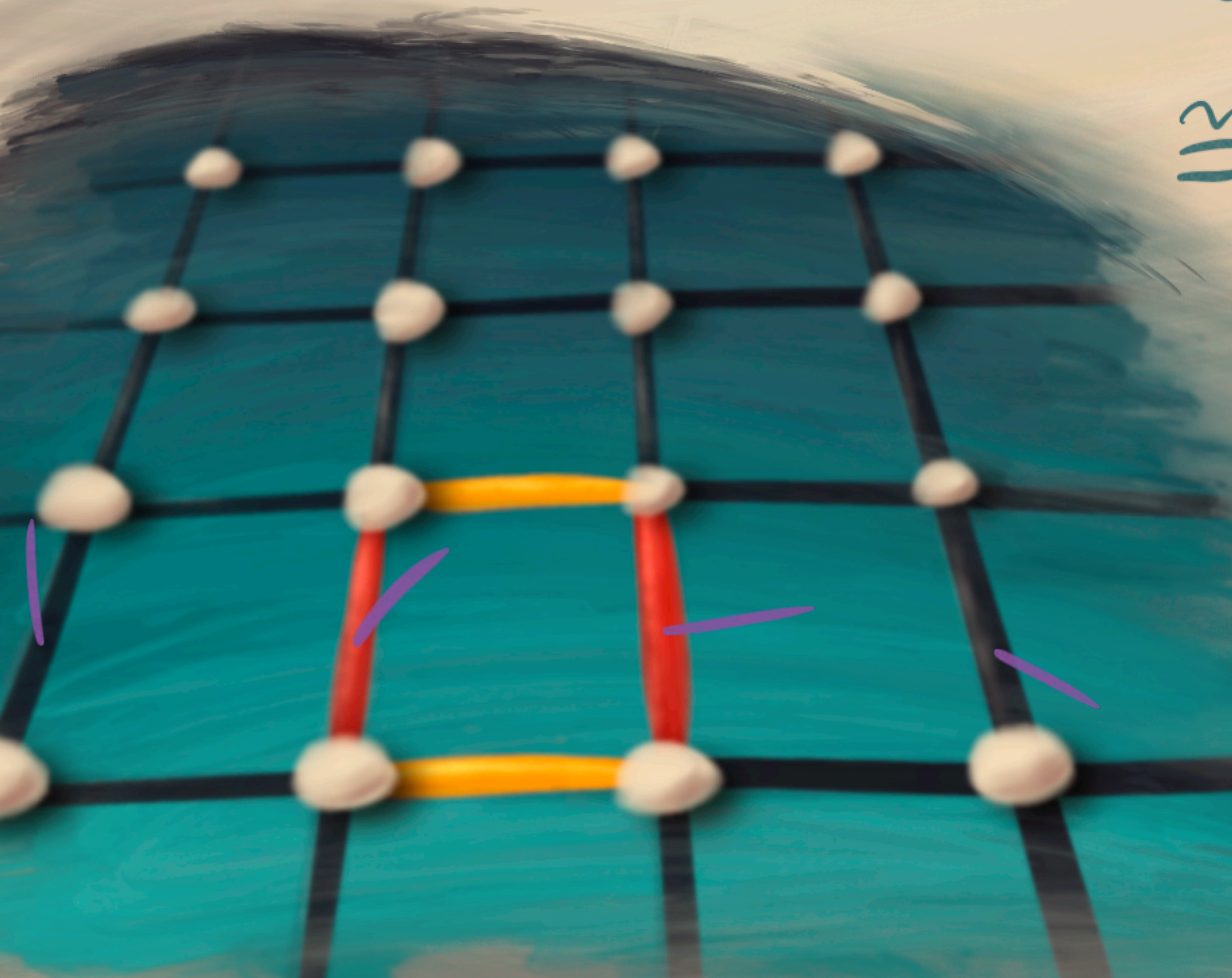
$$\text{Hom}(\mathbb{Z}^2, U(1)) = U(1) \times U(1)$$

Euclidean⁶ crystal



$\text{Hom}(\mathbb{Z}^2, V(1)) \cong \{ \text{Holomorphic Line bundles} \}$

$\cong \text{Jac}(\Sigma)$



Hyperbolic Bloch

$\Gamma \subset \text{Isom}(\mathbb{H}^2)$ nonabelian δ' 8

Theorem

T_γ unitary rep.
 \Rightarrow splits into irreps.

$$\begin{array}{ccc} \Gamma & \xrightarrow{T} & \mathcal{H} \\ \Gamma & \xrightarrow{\rho} & V \end{array}$$

n-dimn |
"multiplet"

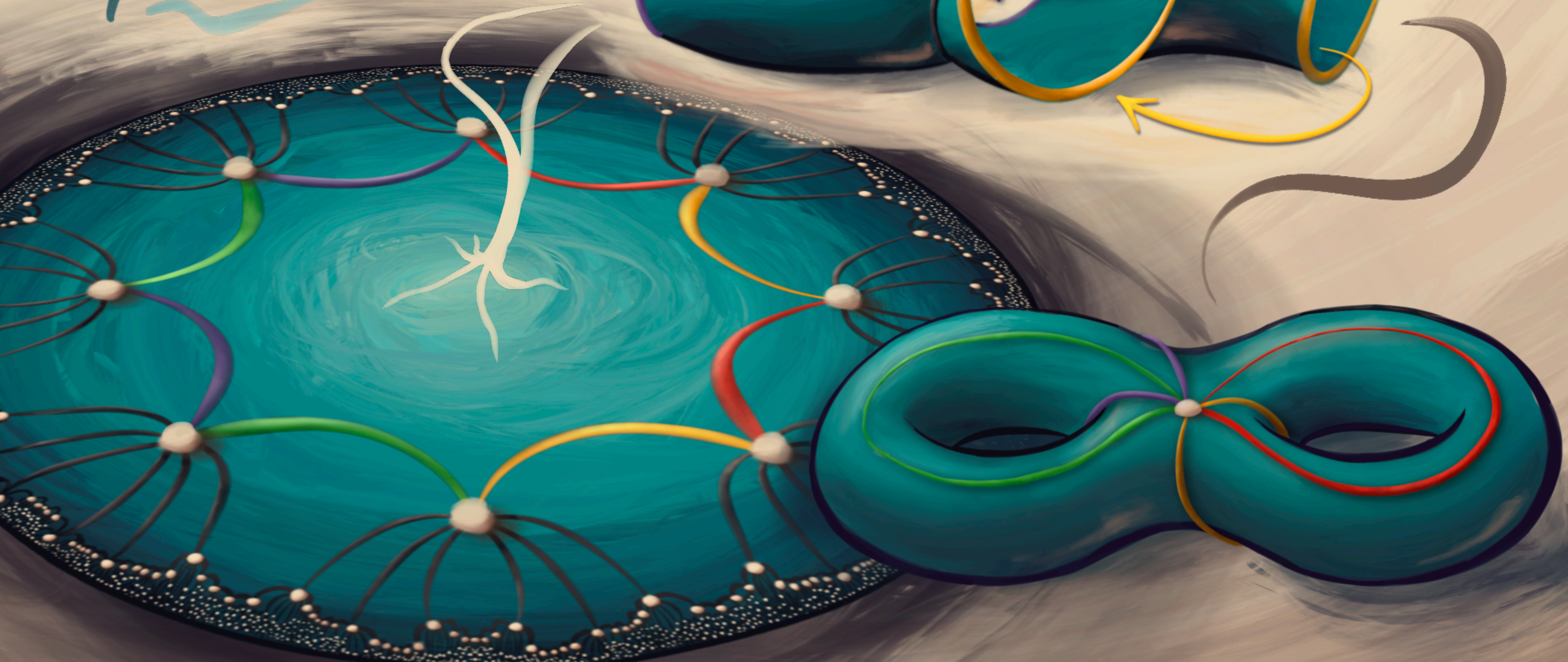
$$\mathcal{H} = \bigoplus_{\text{Hom}_{\text{irr}}(\Gamma, U(n))} \mathcal{H}_\rho$$

T_γ are symmetries $\Rightarrow H: \mathcal{H}_\rho \rightarrow \mathcal{H}_\rho$

Band structure $\text{Spec}(H|_{\mathcal{H}_\rho})$ over $\text{Hom}_{\text{irr}}(\Gamma, U(n))$



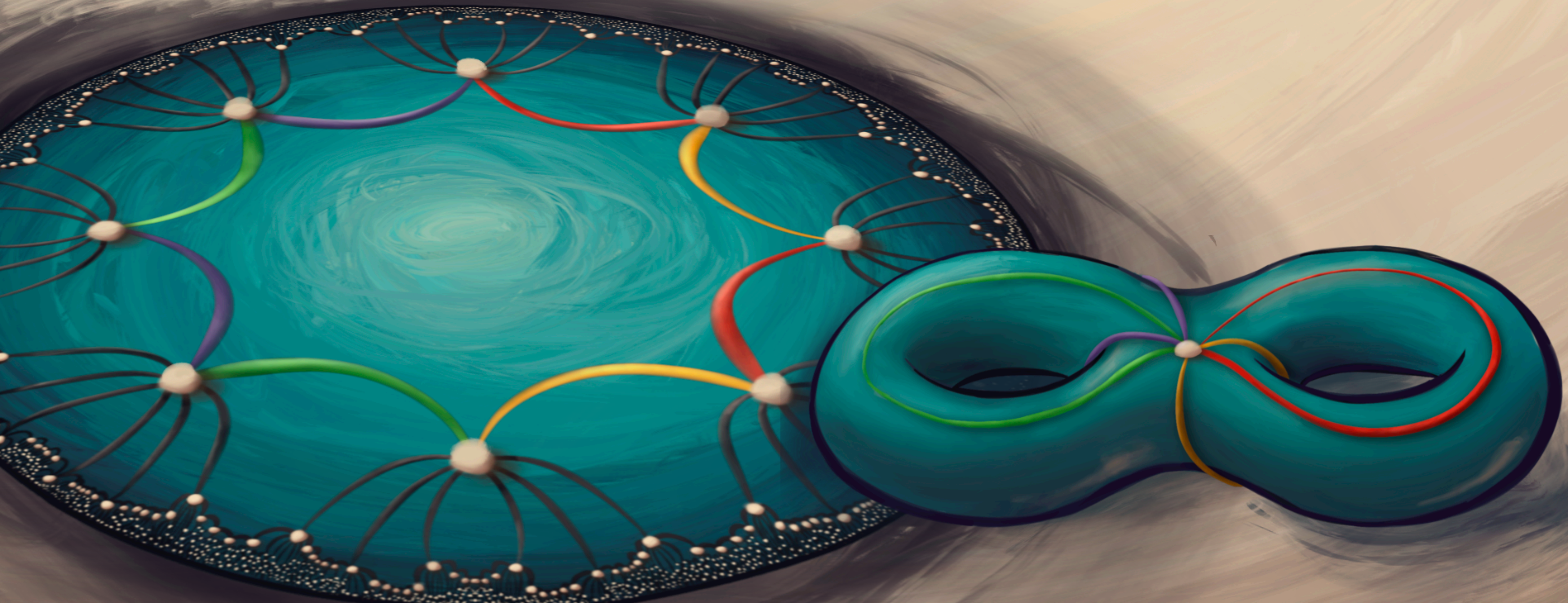
Hyperbolic crystal



Narasimhan-Seshardi

$$\text{Hom}_{\text{irr}}(\Gamma, U(n)) \cong \left\{ \begin{array}{l} \text{stable bundles on } \Sigma \\ \text{rank. } n \quad \text{deg. } 0 \end{array} \right\}$$

$$\mathcal{M}_{d=0}^s$$



Hyperbolic Band Structure

$$H = \bar{\partial}_E^* \bar{\partial}_E + V$$

- Self-Adjoint ✓
- Elliptic ✓
- Compact mfb ✓



$$\mathcal{M}_{n=1, d=0}^s = \text{Jac}(\Sigma)$$



$$\mathcal{M}_{n=2, d=0}^s$$

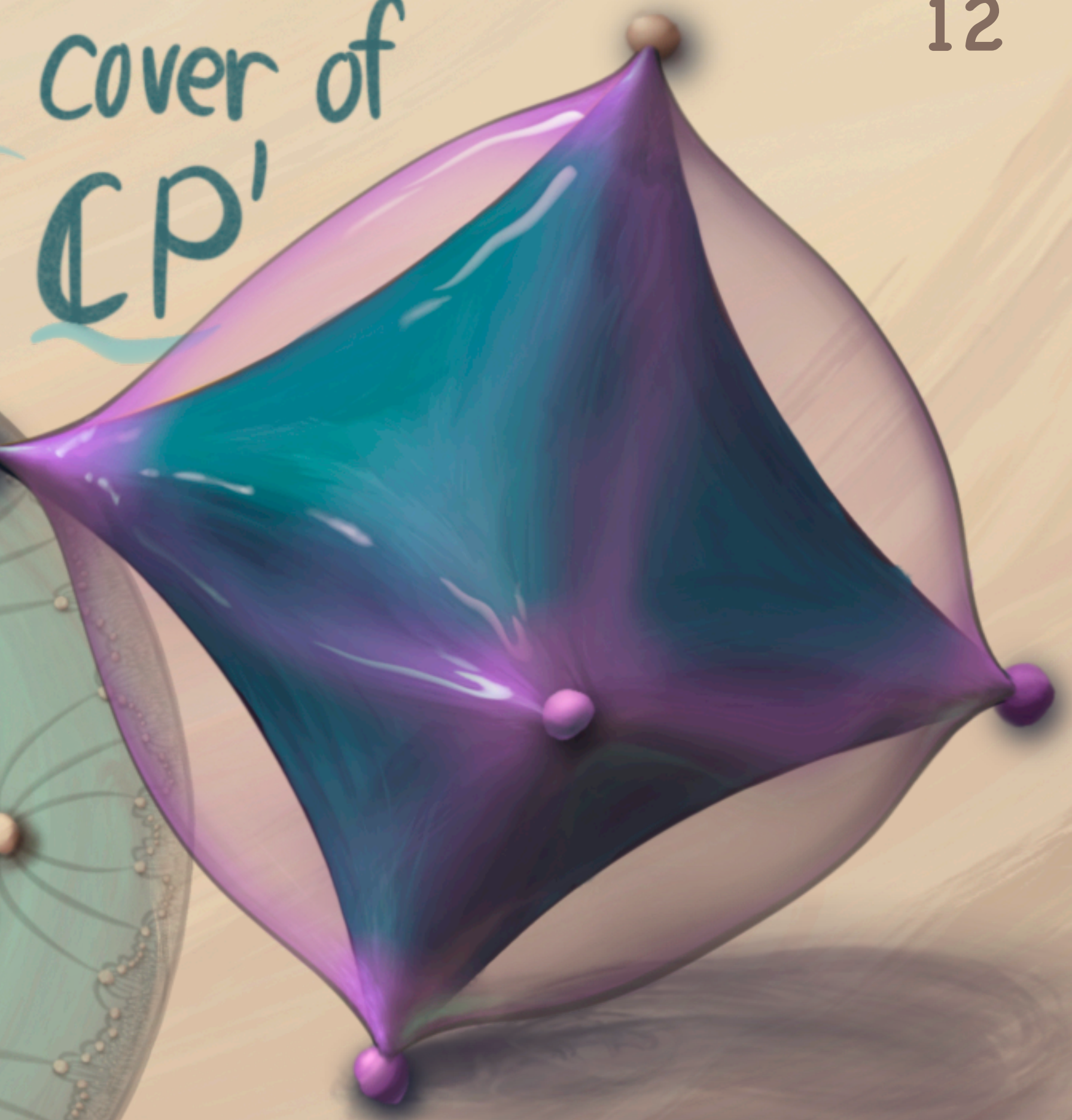
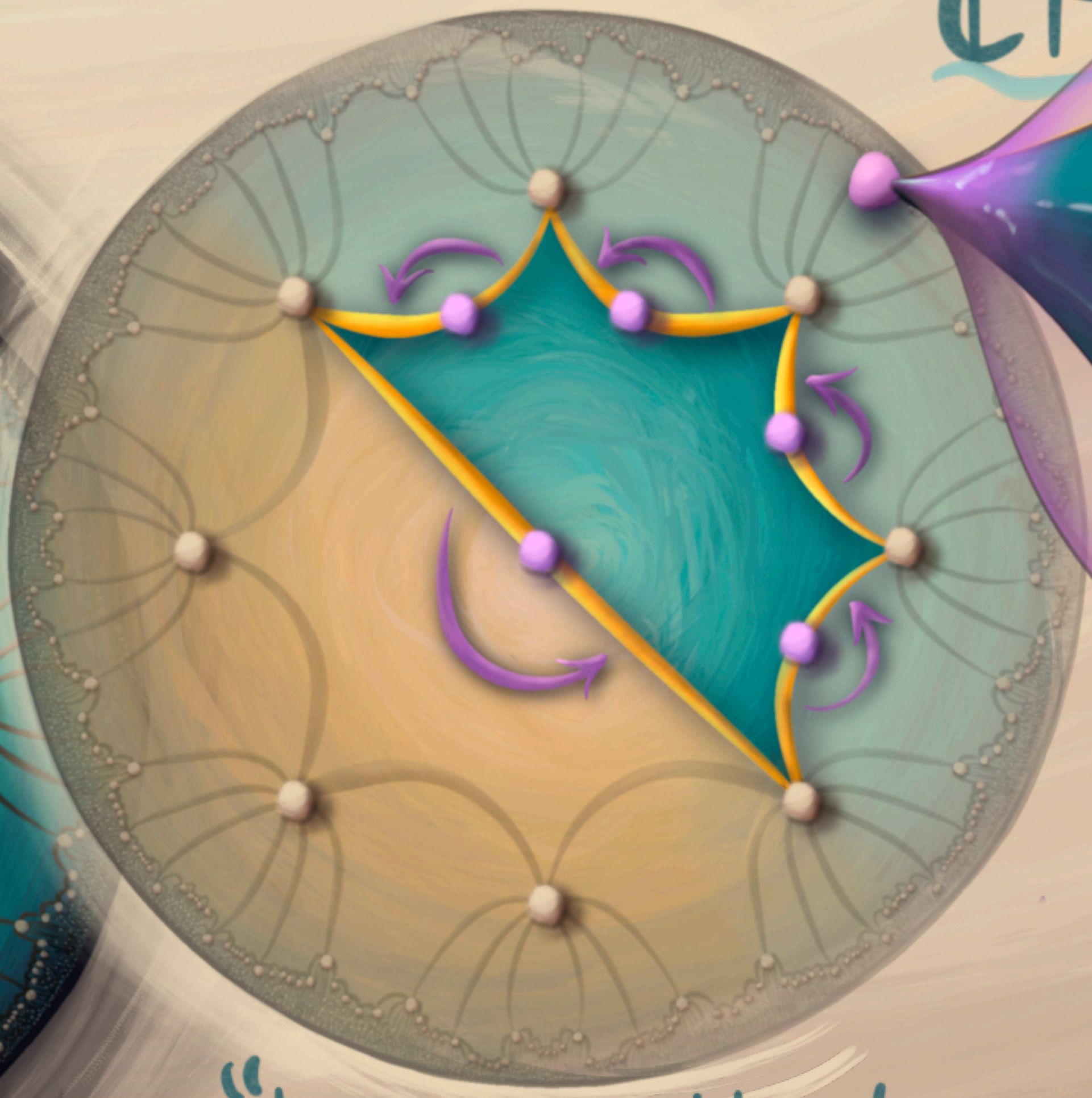
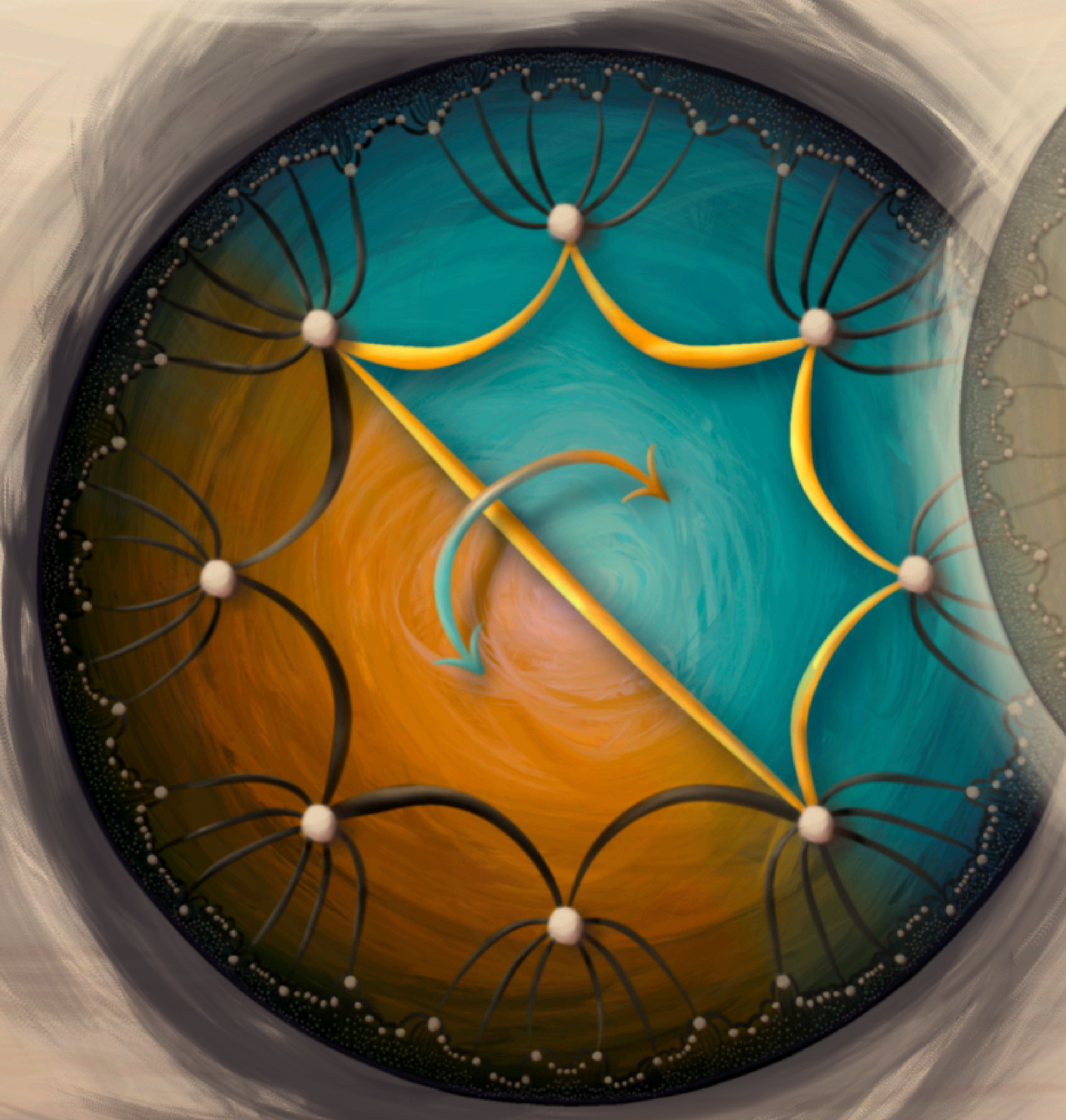


$$\mathcal{M}_{n=3, d=0}^s$$



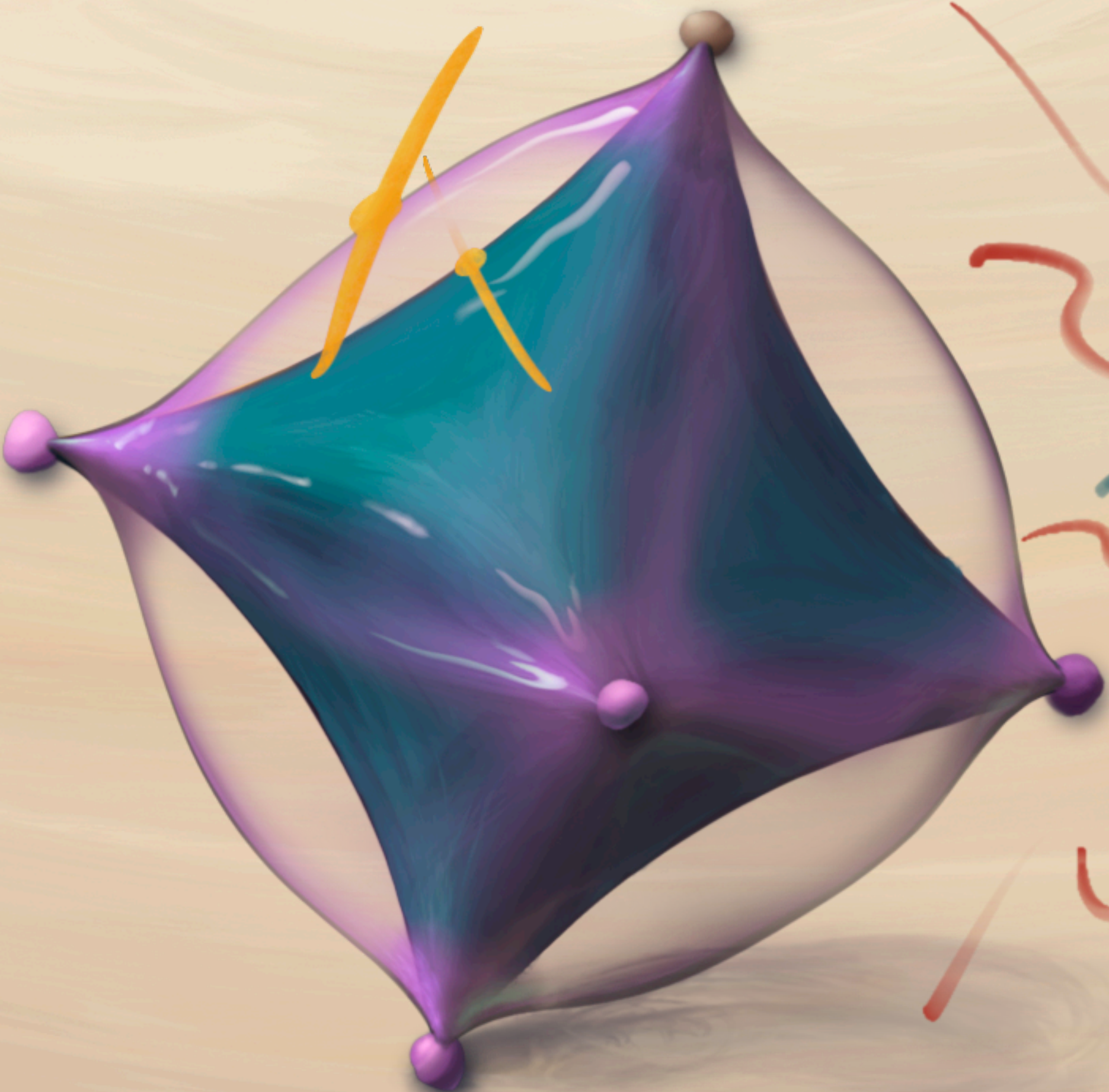
Spatial inversion
symmetry:

Double cover of
 $\mathbb{C}P^1$

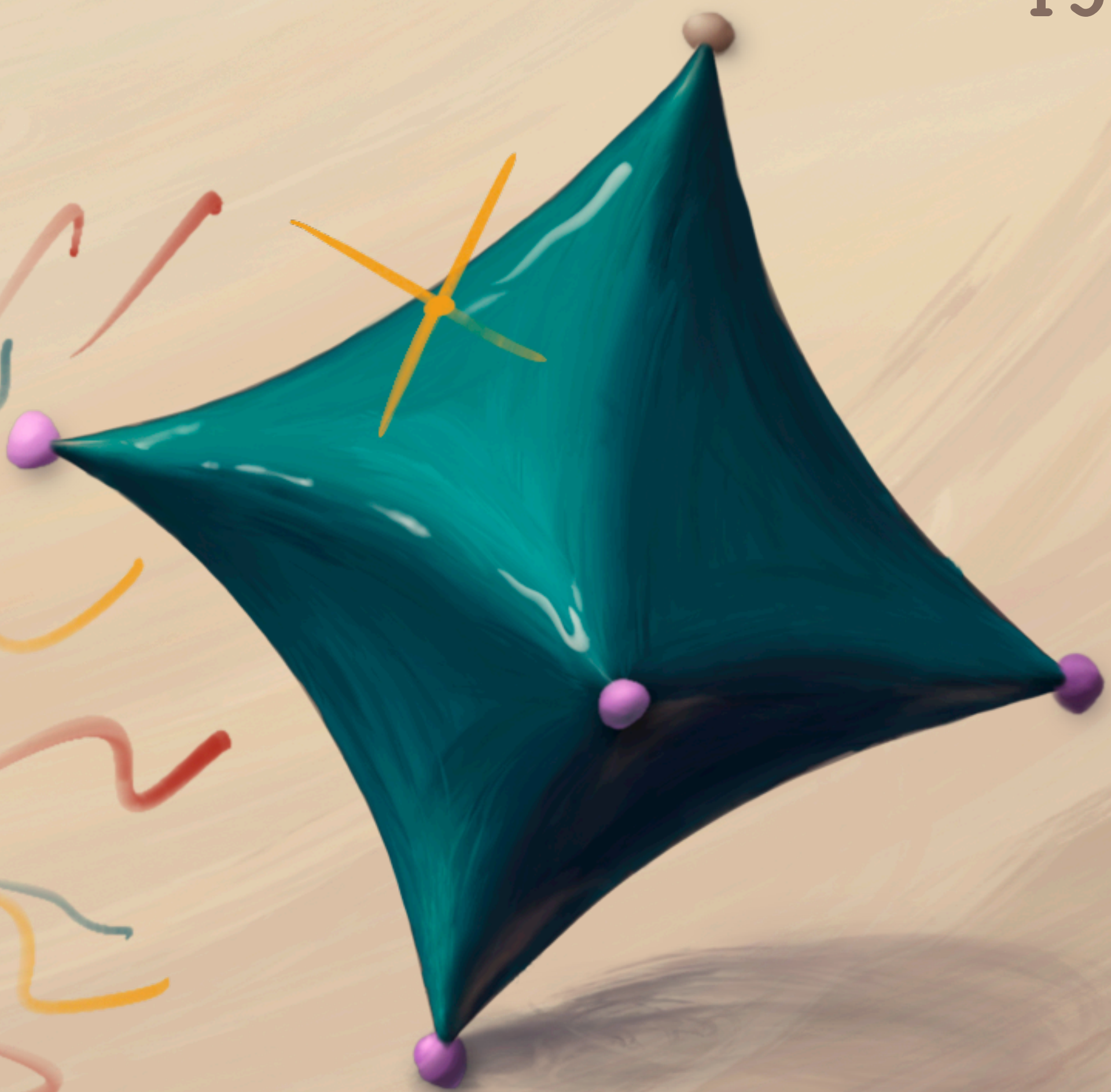


cone points

“hyperelliptic involution”



lattice Σ
L



CP'
 $P * L$

Higgs bundles

Crystal data \Leftrightarrow spectral data ¹⁴

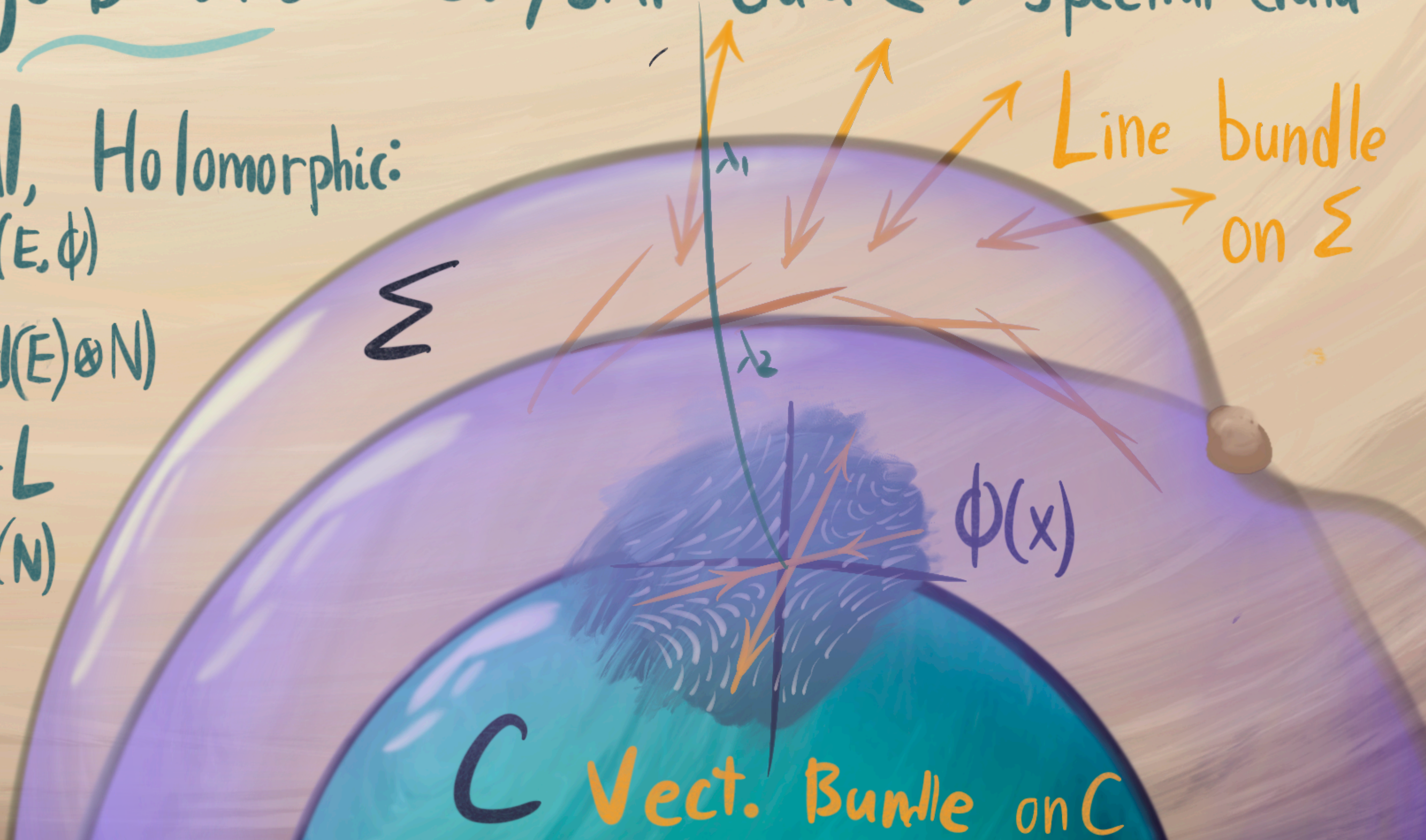
Global, Holomorphic:

pair (E, ϕ)

$$\phi \in H^0(\text{End}(E) \otimes N)$$

$$E = \pi_* L$$

$$\lambda_1, \lambda_2 \in \Gamma(N)$$



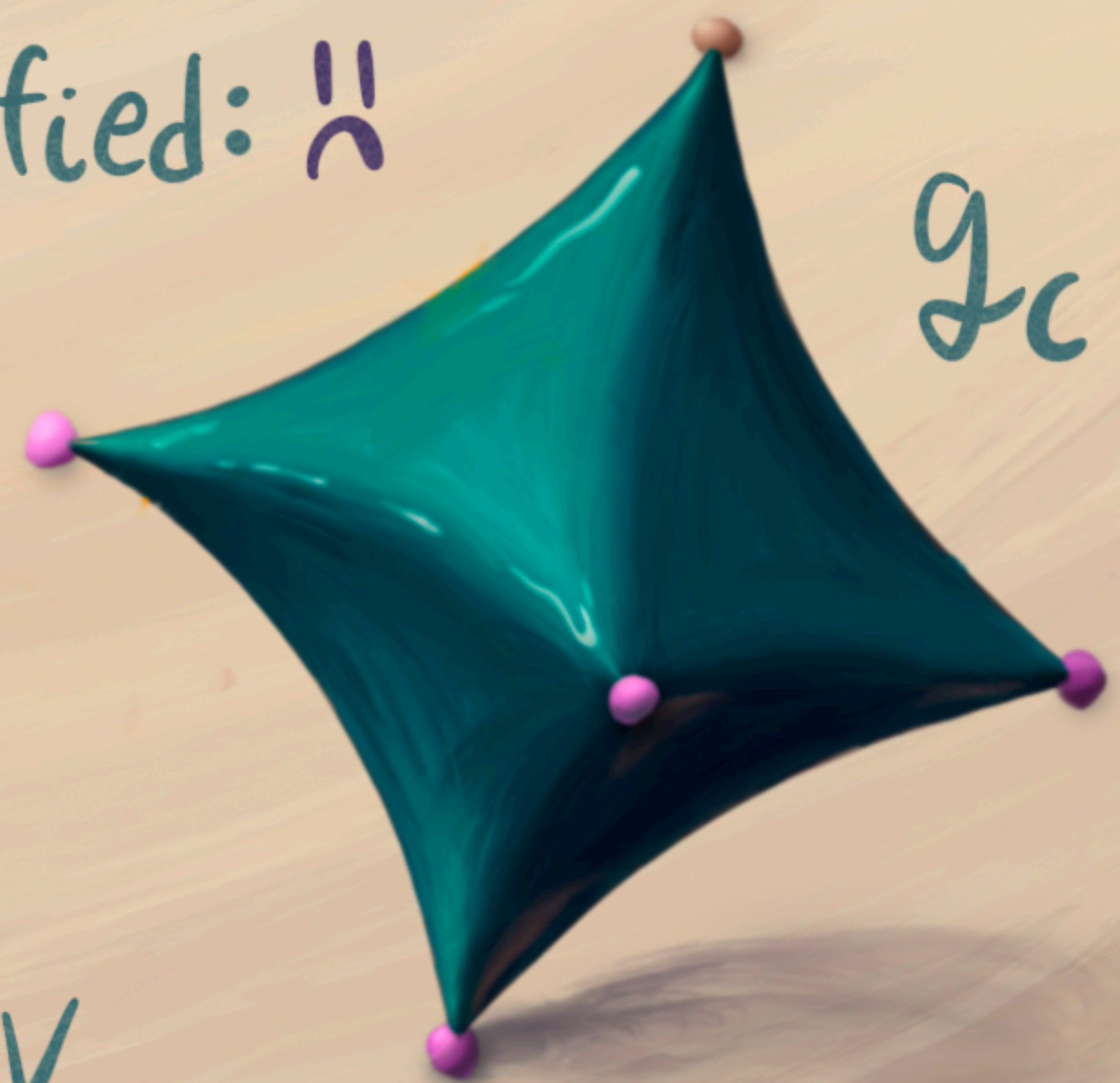
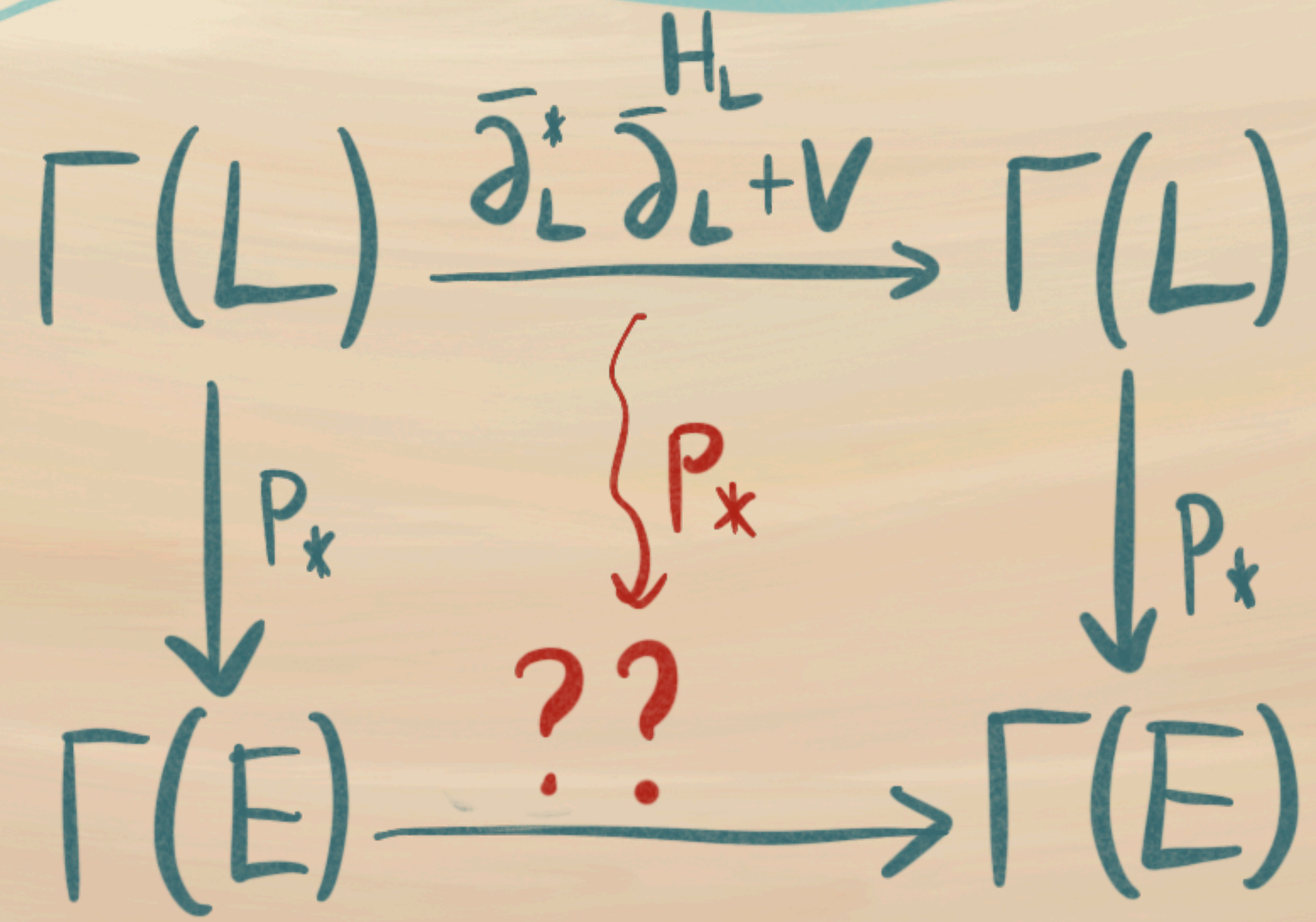
Line bundle on Σ

C Vect. Bundle on C

Pushforward operator

unramified: $P_* H_L = H_E$ ☺

ramified: ☹



g_C singular

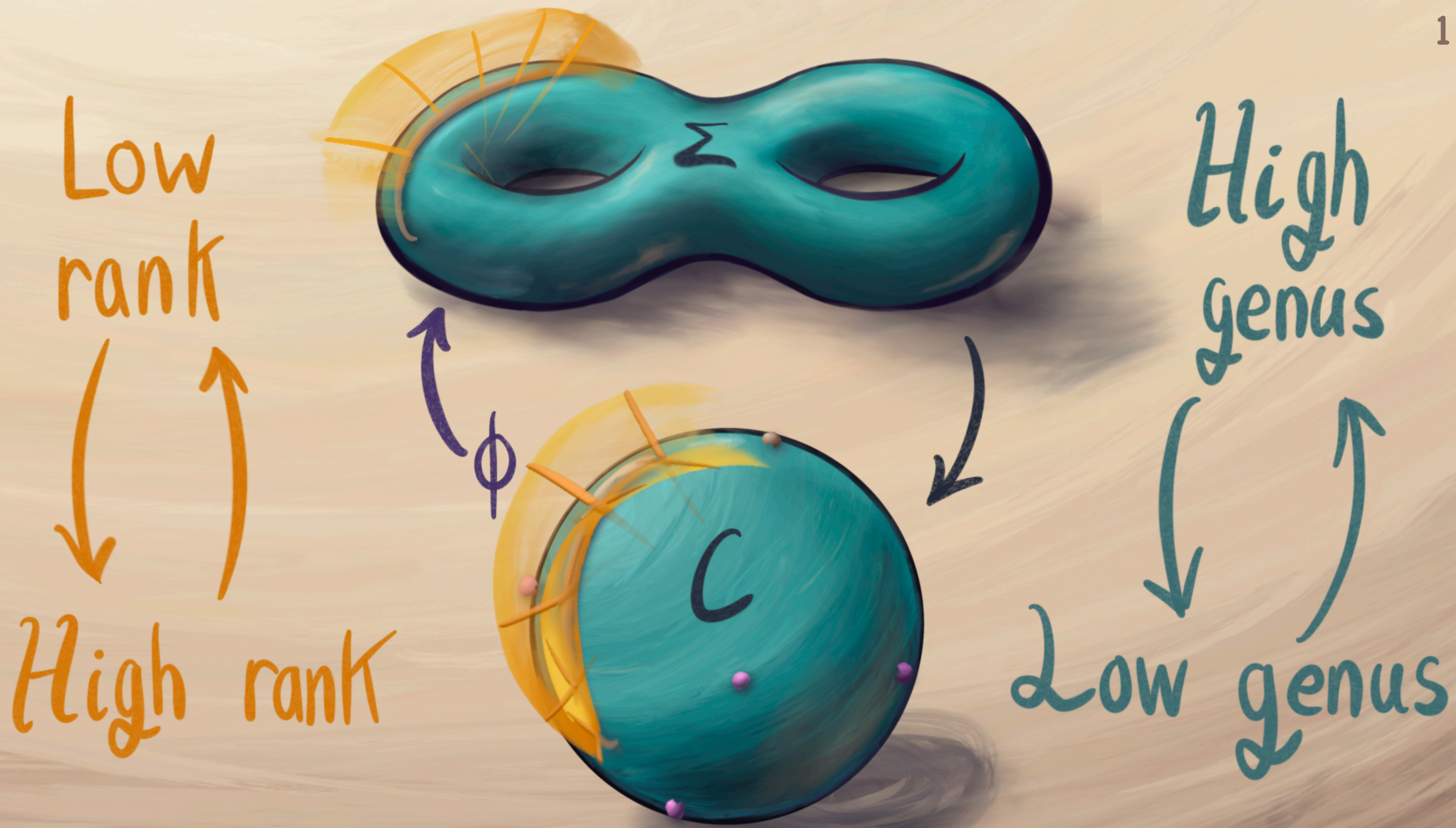
$$P_* \bar{\partial}_L := \bar{\partial}_E$$

$$P_* V = V \cdot Id \text{ when } V \text{ lifts from } C$$

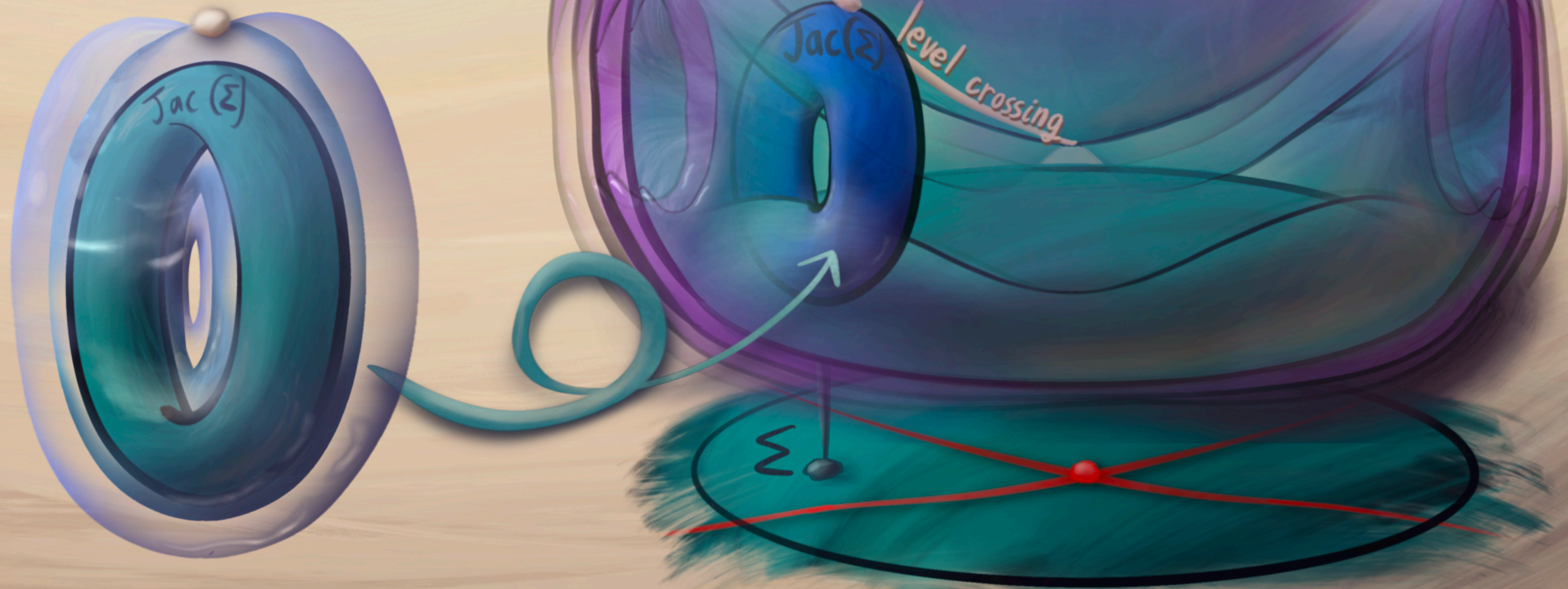
$$P_* \bar{\partial}_L^* = \bar{\partial}_E^*$$

$$\text{when } g_\Sigma = P^* g_C$$

Imposes B.C.s



Universal Band Structure



Complex momenta

in AG-land, we must not suffer \mathbb{R}
 \Rightarrow allow non-Hermitian H

$$\text{Hom}_{\text{irr}}(\Gamma, U(n)) \rightsquigarrow \text{Hom}_{\text{irr}}(\Gamma, GL(n, \mathbb{C}))$$

$$\mathcal{M}^S(\Sigma, \text{rank}=n, d=0)$$

$$\mathcal{M}^{\text{Higgs}}(\Sigma, \text{rank}=n, d=0)$$

Holo.

Flat

$$\bar{\partial}_E$$

$$d + iA$$

$$(\bar{\partial}_E, \phi)$$

$$d + i(A - i(\phi + \phi^+))$$

real momenta



imaginary



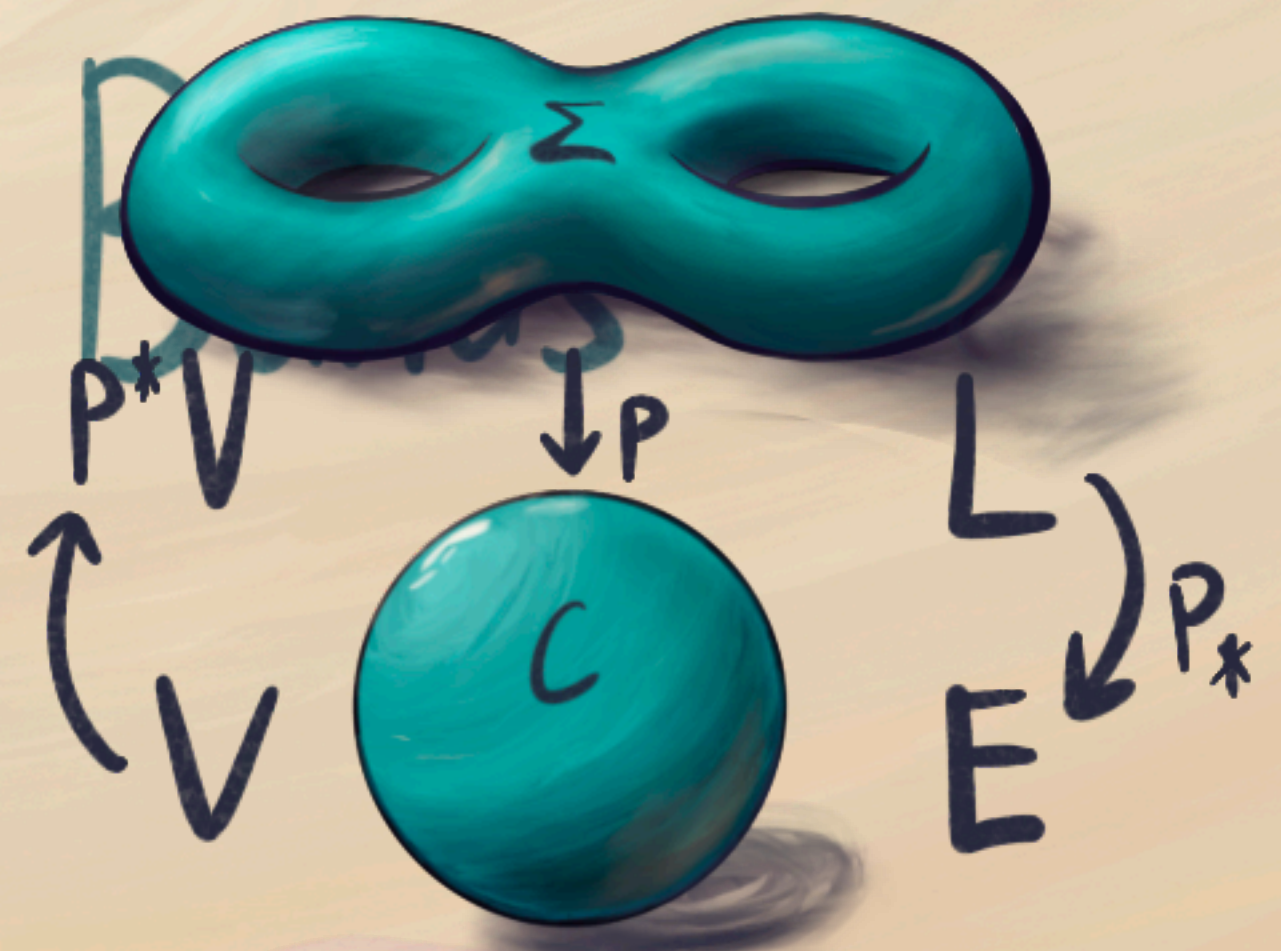
$i\infty$
 $\Im m(k)$

$-\infty$

$\text{Re}(k)$

Complex Momenta

$$(\bar{\partial}_{E^+} \phi + \phi^+)^* (\bar{\partial}_{E^+} \phi + \phi^+) + V$$



Crystal Moduli¹⁹

$$\bar{\partial}_L^* \bar{\partial}_L + p^* V$$

Complex



Real

Daydream:

this diagram commutes

S-Duality

Branes on

Geometric Langlands

S-duality

M^{Higgs}

2-torsion

$SL(2, \mathbb{R})$

fixed pts

Hyperkähler Structure

Seiberg-Witten theory

high-symmetry points

large complex momentum: $\phi \rightarrow \infty$

$N=2, d=4$ SUSY

Gaiotto-Moore-Neitzke: Hyperkähler asymptotics

Hyperbolic Matter

Fourier Transform

interacting States

nonabelian θ -functions

Fractional Quantum Hall

